TEST-04
Modern Algebra

Date- Tue. 7 June 2022

Max mark - 84

Time - 2 hr

Topic- Group Theory

Part-A {Single Correct Question's}

1. The total no. of group homo. from $Z_2 \times Z_2$ to S_3

a. 1 b. 2 c. 9 d. 10

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2. Let $f:(Z,+) \to (\mathbb{C}^*, \cdot)$ be a group homo. define by

$f(n) = \left[\frac{-1+i\sqrt{3}}{2}\right]^n$, Then it's kernal

a. $(4Z,+)$ b $(6Z,+)$ c. $(8Z,+)$ d. $(3Z,+)$

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3. $U(2^k)$ is non-cyclic group for k

a. 1 b. 2 c. 3 d. none of these

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4. Let G be a finite group of order 60. Then

a. G has six 5-ssg b. G has four 3-ssg

c. G has cyclic subgroup of order 6

d. G has a unique element of order 2.

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5. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$, Then

a. $A = A^{-2}$ b. $A = A^{-4}$ c. $A = A^{-5}$ d. $A = A^{-1}$

6. Let order of G is 62. , H is proper subgroup of G then which of the following is true?

a. $H \subset Z(G)$

b. H is cyclic

c. H is cyclic but not cyclic

d. H is unique.

7. Let $\frac{G}{Z(G)}$ is of order 7. Then select correct one

a. order of G is either 7 or 14

b. order of G must be 14

c. order of G must be 7

d. none is correct

8. Consider the two sets $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$ then

a. # of function from A to B 243

b. # of one-one function from A to B 120

c. # of onto function from B to A is 0

d. # of one-one function from A to B is 60.

9. Let $f : (\mathbb{Z}, +) \to (\mathbb{R}^*, \cdot)$ be homomorphism, if $f(2) = \frac{1}{3}$

Then the value of $f(6)$ is

a. $\frac{1}{27}$

b. 27

c. 81

d. $\frac{1}{81}$

10. If a and a^2 both are generator of a cyclic group then order of the group.

a. only Prime

b. only even integer

c. Any odd Positive integer > 1

d. cannot be Prime.

Part-B {Multiple select Question}

1. Let G be a group. H is subgroup of G let $a, b \in G$. Then which of the following is/are true.

a. $Ha = H$ iff $a \in H$
b. $Ha = Hb$ iff $ab^{-1} \in H$

c. $o(H) = o(Ha) = o(aH)$
d. $Ha = aH$ iff $aha^{-1} \in H \ \forall\, h \in H$

2. Let G be a finite group. Then which of the following is/are Incorrect.

a. If $a \in G$, $N(a)$ is subgroup of order $2 \Rightarrow a \in Z(G)$

b. If $a \notin Z(G)$ and $o[N(a)] = 2 \Rightarrow N(a)$ is non-triwial normal subgroup

c. If $a \in Z(G)$ and $o[N(a)] = 2 \Rightarrow N(a)$ is normal subgroup

d. Both a and b.

3. If $o[G/Z(G)] = p^2$, Then which of the following is/are true.

a. $G/Z(G)$ is cyclic

b. $G/Z(G)$ is abelian but not cyclic

c. Every proper subgroup of $G/Z(G)$ is cyclic

d. G is non-abelian.

4. In S_n, $n=6$, let $G_1 = \langle (123) \rangle$, $G_2 = \langle (123)(456) \rangle$
$G_3 = \langle (132)(465) \rangle$ Then

a. G_1, G_2, G_3 are isomorphic

b. G_1 is isomorphic to G_3 but not G_2

c. $G_1 \cap G_2$ is isomorphic to $G_2 \cap G_3$

d. $G_1 \cap G_2$ is isomorphic to Z_3

5. If $o(G) = 36$, $H < G$, $o(H) = 4$ Then

a. $H \subset Z(G)$ b. $H \trianglelefteq G$

c. $H = Z(G)$ d. H is abelian

6. Which of the following is/are true.

a. G is abelion iff $G = Z(G)$

b. G is abelian iff $N(a) = G \ \forall \ a \in G$

c. $Z(G) = \bigcap_{a \in G} N(a)$

d. G is Cyclic if $G = Z(G)$

8. Let G_1 is cyclic group of order 21, G_2 is non-abelion group of order 125, G_3 is Dehidral group of order 8 Then

a. number of 7-ssg in $G_1 \times G_2 \times G_3$ is unique

b. Center of $G_1 \times G_2 \times G_3$ is abelian but not cyclic

c. $G_1 \times G_2 \times G_3$ has an element of order 8.

d. 2-ssg is normal in $G_1 \times G_2 \times G_3$.

9. Let $G = Z_4 \times Z_4$, $H = \{(0,0), (2,0), (0,2), (2,2)\}$ and $K = \langle (1,2) \rangle$, Then select the correct

a. $\frac{G}{H} \approx Z_4$,   b. $\frac{G}{K} \simeq Z_4$ c. $\frac{G}{H} \simeq Z_2 \times Z_2$ d. $\frac{G}{K} \simeq Z_2 \times Z_2$

10. In S_7, $\alpha = (12345)(67)$, $\beta = (14253)(67)$ $\gamma = (1243)(56)$ and they are shown as:

(a) $\alpha^7 = \beta^3$ (b) $\alpha^5 = \gamma^2$ (c) $\beta^3 = \gamma^2$

Then which of the following is/are true.

a. a, b   b. a, b, c

c. b, c   d. none.

Part-C { Numerical Answer Type }

1. If G is a finite group, and $a \in G$ Then $a^{o(G)} =$ ____.

2. Let $a = 5^2$, $b = 20$, $c = 15$, $d = 20$ Then value of $\text{Lcm}\{\phi(d), \tau(c)\} \cdot \text{Gcd}\{\sigma(b), \omega(a)\} =$ ____

3. The Automorphism of cyclic Group is always ____

4. The number of Distinct subgroup of Z_{999} ____

5. The value of $\phi[\sigma\{\tau(\omega(100))\}]$ is ____

6. How many normal subgroup in D_{2025} ____

7. The number of Automorphism $f : D_{20} \to D_{20}$ is ____

8. The order of $\overline{10}$ in $U(37)$ is ____

9. The Identity element of Group $(P(N), \Delta)$ ____

10. Total number of conjugate class in S_5 ____

11. Let $G = Z_2 \times U(3) \times K_4$, and $a \in G$, How many elements satisfy the condition $a^2 = e$, e is Identity element of G.

12. If H and K are subgroup of G, then HK is a subgroup of G iff ____

.– Best Wishes from Vivek Sahu –.